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Mathematical Solution
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PROBLEM

What is the derivative of $\sin(x)$, and how does this limit relate to it?

We're going to find the derivative of the sine function, which is a fundamental concept in calculus. This will involve understanding limits and how they relate to derivatives.

VERIFIED ANSWER

$$\cos(x)$$

Step-by-Step Solution

1 Recall the Definition of a Derivative

The derivative of a function $f(x)$ is defined as the limit of the difference quotient:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

2 Apply the Definition to $\sin(x)$

We'll use the definition to find the derivative of $\sin(x)$:

$$\lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

3 Use Trigonometric Identities

We can use the angle addition formula for sine to rewrite $\sin(x+h)$:

$$\sin(x+h) = \sin(x)\cos(h) + \cos(x)\sin(h)$$

4 Substitute the Angle Addition Formula

Substituting the angle addition formula into the limit, we get:

$$\lim_{h\to 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h}$$

5 Simplify the Expression

We can simplify the expression by canceling out the $\sin(x)$ term:

$$\lim_{h\to 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h} = \lim_{h\to 0} \frac{\cos(x)\sin(h)}{h}$$

6 Evaluate the Limit

The limit can be evaluated by recognizing that $\lim_{h\to 0} \frac{\sin(h)}{h} = 1$:

$$\lim_{h\to 0} \frac{\cos(x)\sin(h)}{h} = \cos(x) \cdot 1 = \cos(x)$$

Therefore, the answer is:

$$\cos(x)$$